Exercises Module 2

Corrections

Exercise 2.1

The shear force is equal to the shear stress on the surface element multiplied by the surface area. To identify the relevant component contributing to the shear force, we can use the general formula for the shear force F_{shear}:

$$F_{shear} = -\tau_{jk}\big|_j(\hat{\boldsymbol{n}}_{\boldsymbol{i}} \cdot \hat{\boldsymbol{j}})\hat{\boldsymbol{k}}A_i$$

with the direction of flow k (unit vector $\hat{\boldsymbol{k}}$) and the direction of momentum transport j (unit vector $\hat{\boldsymbol{j}}$). $\hat{\boldsymbol{n}}_{\boldsymbol{i}}$ is the surface normal vector of the considered surface A_i.

From Newton's law of viscosity, we know that shear stress as a momentum flux in direction j originates from a non-zero velocity gradient $\frac{dv_k}{dj}$ of v_k in direction of j.

If fluid is flowing over the surface of a solid body, the no-slip condition has to be fulfilled at the surface. If the body is not in motion, this implies a velocity of zero right at the surface and on the whole surface. Therefore, there can be no gradient of the velocity along the directions moving on the surface, but only in the directions moving away from the surface element. For example, the velocity on the surface does not vary in z or θ in the case of a solid cylinder, because it needs to be zero everywhere on the cylinder.

With that in mind, we can derive the terms for the shear force on the indicated surface elements:

a) $\boldsymbol{dF_{shear}} = -\tau_{r\theta}|_{r=R}Rd\theta \times dz \times (\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{r}})\hat{\boldsymbol{\theta}} = -\tau_{r\theta}|_{r=R}Rd\theta\, dz\, \hat{\boldsymbol{\theta}}$

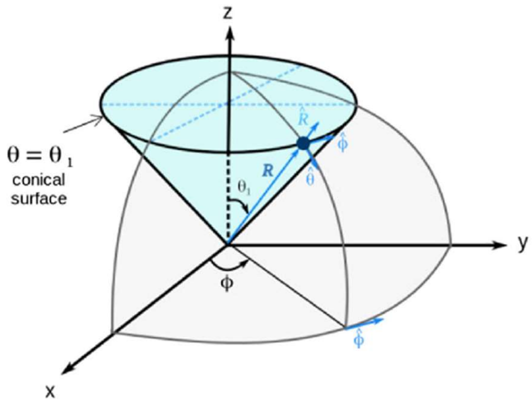
b) $\boldsymbol{dF_{shear}} = -\tau_{rz}|_{r=R}Rd\theta \times dz \times (\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{r}})\hat{\boldsymbol{z}} = -\tau_{rz}|_{r=R}Rd\theta\, dz\, \hat{\boldsymbol{z}}$

c) $\boldsymbol{dF_{shear}} = -\tau_{r\theta}|_{r=R}Rd\theta \times R\sin\theta d\phi \times (\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{r}})\hat{\boldsymbol{\theta}} = -\tau_{r\theta}|_{r=R}R^2\sin\theta\, d\theta d\phi\, \hat{\boldsymbol{\theta}}$

d) $\boldsymbol{dF_{shear}} = -\tau_{r\phi}\big|_{r=R}Rd\theta \times R\sin\theta d\phi \times (\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{r}})\, \hat{\boldsymbol{\phi}} = -\tau_{r\theta}|_{r=R}R^2\sin\theta\, d\theta d\phi\, \hat{\boldsymbol{\phi}}$

e) $\boldsymbol{dF_{shear}} = -\tau_{\theta r}|_{\theta=\alpha}dr \times (r\sin\alpha\, d\phi) \times (\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{\theta}})\hat{\boldsymbol{r}} = -\tau_{\theta r}|_{\theta=\alpha}r\sin\alpha\, drd\phi\, \hat{\boldsymbol{r}}$

The coordinate system in case e) is a spherical coordinate system. The cone is placed within a sphere with the origin of the cone being at the origin of the sphere. θ and ϕ are chosen in a similar way as for a sphere.



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